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NONSTATIONARY HEAT EXCHANGE IN THE TRACHEA OF HUMAN LUNGS

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Calculations of the heat exchange in the trachea of human lungs have been carried out. Unlike other works in this field, in the present work the authors have taken account of the nonstationarity of the processes, which is related to the alternation of inhalations and exhalations and the variability of the air flow rates. Nonstationary changes in the temperature of trachea walls in quiet breathing have been calculated in different cases.

The lungs contain the airways, i.e., the trachea (the trachea is preceded by the nasopharynx and the pharynx), which branches into two bronchi with their subsequent branchings forming the bronchial tree of the right and left lungs. Small bronchial tubes (alveolar ducts) contain thin-walled sacs, i.e., alveoli, on the lateral surfaces through which the gas exchange between oxygen and carbon dioxide and blood occurs. The alveoli form a spongy mass filling the entire thoracic cavity except for the places occupied by the heart, blood vessels, airways, and esophagus.

In the airways (similarly to the nasopharynx and the pharynx), air is heated or cooled depending on its inlet temperature. Furthermore, the air becomes saturated with moisture (or excess moisture condenses on channel walls); the heat of evaporation is absorbed or the heat of condensation is released. In such a manner, mass exchange between the water vapor (steam) and the air is carried out. This mass exchange is considered below along with heat exchange.

Heat exchange (similarly to mass exchange) in the lungs is of a nonstationary character in view of the nonstationarity of inhalation and exhalation. Heat exchange in the airways of the lungs as a stationary process has been considered earlier in detail in many works (for example, [1–4]). In the present work, we attempt to make further refinements in considering heat exchange (and mass exchange) in the lungs as a nonstationary process.

The conjugate nonstationary problem of heat and mass exchange of moist air in the trachea channel can be solved numerically in a two-dimensional formulation. One must take into account changes in the thermal state of the wall under conditions of flow varying in direction and temperature and moisture content varying with time. Leaving such a solution for the future, in the present paper we will employ a one-dimensional approximation with the aim of revealing, in general terms, the influence of the nonstationary processes of inhalation and exhalation.

The adopted one-dimensional approximation enables us to write the nonstationary equation of heat exchange in the trachea channel in the following form:

$$\rho C_p \frac{\partial \vartheta}{\partial \tau} + \rho C_p w \frac{\partial \vartheta}{\partial x} = -\alpha \vartheta \frac{4}{d} .$$

Here we disregard metabolic heat release, since it is small in the lungs [1–3].

After the separation of the variables τ and x, the nonstationary equation of heat exchange (at t_w = const) is reduced to the quadratures

$$\vartheta (x, \tau) = \vartheta_{in} \exp \left(- 4\mathrm{St}_{min} \frac{x}{d} \right) \exp \left(- 4\mathrm{St}_{min} \frac{w_{max}}{d} \int_0^{\tau} (1 - \varphi (\nu))\, d\nu \right), \qquad (1)$$

where $\vartheta_{in} = t_{in} - t_w$, t_{in} being the inlet temperature.

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Analogously we write the nonstationary equation of mass exchange for the steam, which will involve the mass-transfer coefficient β instead of the heat-transfer coefficient α. The solution of the mass-exchange equation (at Y_w = const) has the form

$$Z(x, \tau) = Z_{in} \exp\left(-4\mathrm{St}_{D\min} \frac{x}{d}\right) \exp\left(-4\mathrm{St}_{D\min} \frac{w_{\max}}{d} \int_0^\tau (1 - \varphi(\nu))\, d\nu\right), \tag{2}$$

here $Z_{in} = Y_{in} - Y_w$, Y_{in} is the relative partial pressure of the steam at the inlet.

It is natural that the above nonstationary equations and their solutions hold for both inhalation and exhalation.

As the air flow rate changes, each new flow rate is established along the channel instantaneously, in practice (with the velocity of sound). Solutions (1) and (2) hold within a short time interval after replacement of the old air by new air entering in inhalation (or going out in exhalation) on the considered portion of the trachea.

The heat and mass exchange is calculated simultaneously. To take account of the variability of t_w and Y_w we will employ the zonal method of calculation with constant (average) values of t_w and Y_w inside narrow zones along the channel length. In view of the degree of approximation of the formulation of the problem, it will suffice to take the same physical quantities for the mass as for water.

The condensation of the vapor on the wall or, conversely, evaporation from the film of the liquid phase from the wall surface can substantially influence the supply or removal of heat at the air–liquid film boundary. These processes also influence the vapor content of the air flow in terms of the boundary value of Y_w. The relation between the mass exchange and the flow temperature can be direct in the case of formation of a mist in the air (release of heat) or, conversely, the evaporation of droplets (absorption of heat), which is not observed in the airways of the lungs. Also, we note that the velocities of the transverse vapor flow to the walls are small as compared to the longitudinal velocities of moist air; therefore, we can disregard the distortions of the gas boundary layer. As a consequence we can assume that the coefficients of heat and mass transfer are related by the well-known formula of Bowen

$$\frac{\alpha}{\beta} = \frac{\lambda}{D} f(\mathrm{Le}),$$

where $f(\mathrm{Le}) = \mathrm{Le}^{0.4}$ is the function of the Lewis number.

Thus, we have

$$\beta = \frac{D\alpha}{\lambda \mathrm{Le}^{0.4}}. \tag{3}$$

In the case of phase transition on the liquid film, the heat flux is released (condensation) or absorbed (evaporation):

$$q_\beta = r\beta \frac{M_v P}{R\bar{T}} (Y - Y_w).$$

The total quantity of heat given (received) by the wall is determined by the algebraic sum

$$q_\Sigma = q_\alpha + q_\beta;$$

here q_α is the heat flux caused by convective heat transfer. It follows that the total heat-transfer coefficient determining the thermal state of the liquid film on the wall is also determined by the algebraic sum

$$\alpha_\Sigma = \alpha + r\beta \frac{M_v P}{R\bar{T}} \frac{(Y - Y_w)}{(t - t_w)}. \tag{4}$$

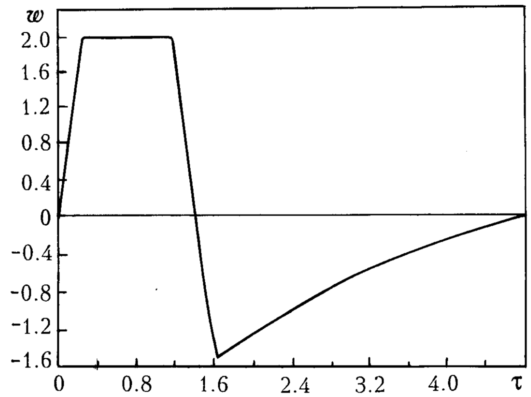


Fig. 1. Change in the velocity of air in the trachea during one interval of breathing (inhalation–exhalation). w, m/sec; τ, sec.

The value of the total coefficient of heat transfer α_Σ is frequently determined by the second term (responsible for the phase transition), which is comparable to or even exceeds α depending on the initial conditions (moisture content and temperature of the incoming air).

Since the liquid film on the walls is very thin (its thickness is no more than 50–200 μm), we will assume that the film temperature coincides, in practice, with the wall temperature t_w. In determining the relative partial pressure of the vapor at the wall Y_w as a function of the absolute temperature $T_w = t_w + 273$, we will employ the following formula:

$$\ln P_{v,w} = K_1 + \frac{K_2}{T_w} + \frac{K_3}{T_w^2}, \tag{5}$$

where K_1 = 11.872, K_2 = –3836.2, and K_3 = –217,060.3. This formula approximates the tabulated values of pressure (in 10^5 Pa) for the saturated steam in the temperature range 273–373 K [5].

In accordance with the ideas of otorhinolaryngologists, it had been assumed earlier that the nasopharynx plays a dominant role in heating or cooling of inhaled air. In accordance with the results of more recent investigations of pulmonologists, it is assumed that, conversely, the distal portions of the airways are of paramount importance in conditioning the air [2]. Let us calculate heat and mass exchange for the trachea, assuming that air is directly supplied to the inlet of the trachea and escapes the nasopharynx or the temperature and moisture content of the air after the nasopharynx are known.

In calculations of the heat exchange in the airway channels, one usually assumes that the temperature of the wall is equal to the temperature of the blood (see, for example, [4]). The possible dependence of the velocity of the air at rest in the trachea in inhalation and exhalation is presented in Fig. 1. In inhalation whose total duration is 1.4 sec, the flow rate of the air is nearly constant and it corresponds to the velocity in the trachea w_{max} = 2 m/sec. The duration of the exhalation with a variable flow rate of the air is 3.2 sec; in the process of exhalation we have w_{max} = –1.5 m/sec. The total time of an inhalation–exhalation cycle is equal to 4.6 sec. At the beginning and at the end of inhalation, as the air velocity changes, we have taken the values of $\varphi(\tau)$ that correspond to the linear law. To describe the exhalation curve use is made of both the linear and quadratic laws.

The trachea, whose diameter and length are respectively 0.018 and 0.12 m according to the data of mean-statistical measurements [6], was subdivided (as has already been mentioned) into narrow portions along the longitudinal coordinate; we considered t_w and Y_w to be constant within these portions (the general accuracy of calculation turned out to be sufficient, see below). Changes in t and Y in the portions were determined from formulas (1) for heat exchange and (2) for mass exchange respectively. The literature data on the coefficient of heat transfer α in the trachea are contradictory; very high values are recommended, among others, in [7]. In our calculations, we took α = 10 W/(m^2·deg) as the one most corresponding to the calculations from the formula of the Nusselt number as a function of the Reynolds number in the turbulent regime of flow and to the literature data of model experiments for the trachea. In calculating the wall temperature, we took account, in several approximations, of the removal of heat to the

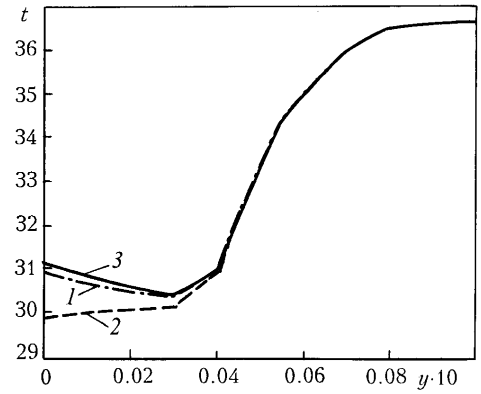


Fig. 2. Temperature distributions in the depth of the trachea wall for $x = 10^{-2}$ m and at an inlet temperature of air of 20°C and a relative humidity of 76% (thirtieth interval of breathing) for the instants of time: 1) 0.2 sec from the beginning of inhalation; 2) 1.2 sec from the beginning of inhalation; 3) 4.4 sec from the beginning of inhalation (exhalation). t, °C; y, m.

body mass or the supply of it from the mass and especially the consumption of heat in the nonstationary regime by heating the wall layer adjacent to the flow (the liquid film and the wall were assumed to be a unit) or the liberation of heat in cooling the wall. The fact is that the heat capacity per unit volume of the mass is more than 1000 times higher than that of the air. Therefore, the consumption of heat by nonstationary heating or cooling is relatively large. To also take into account the nonstationary heating (or cooling) and removal (or supply) of heat by conduction of the mass of the surrounding tissue we can employ the solution of the corresponding problem of mathematical physics. Since the difference in heat capacities per unit volume of the air and the body mass is very great, the nonstationarily disturbed layer of the mass must be thin. Therefore, in calculating the nonstationary processes, the trachea channel will be considered to be a flat slot surrounded by a semiinfinite mass. For the latter having the coordinate temperature distribution $t_{m0}(y)$ at the initial instant of time and then subjected to thermal action on the source side of the air with a temperature t_0 and a heat-transfer coefficient α_Σ, the coordinate and time temperature distribution in the mass $t_m(y, \tau)$ is determined by the formula [8]

$$\vartheta_m(y,\tau) = t_m(y,\tau) - t_0 = \frac{1}{2\sqrt{\pi a_m \tau}} \int_0^\infty \left\{ \vartheta_{m0}(\eta) \left[\exp\left(-\frac{(\eta - y)^2}{4a_m\tau} \right) + \exp\left(-\frac{(\eta + y)^2}{4a_m\tau} \right) \right] - \right.$$

$$\left. - 2\frac{\alpha_\Sigma}{\lambda_m} \exp\left(-\frac{\alpha_\Sigma \eta}{\lambda_m} \right) \exp\left(-\frac{(\eta + y)^2}{4a_m\tau} \right) \int_0^\eta \vartheta_{m0}(\zeta) \exp\left(\frac{\alpha_\Sigma \zeta}{\lambda_m} \right) d\zeta \right\} d\eta\,, \quad (6)$$

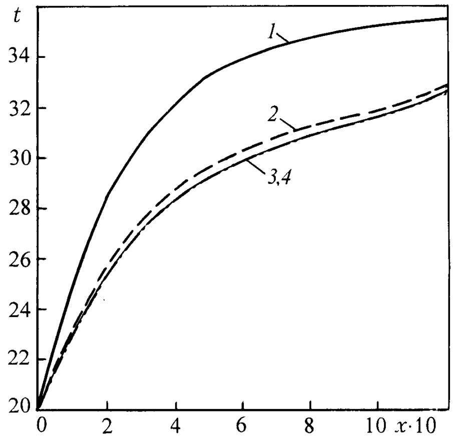
where $\vartheta_{m0}(y) = t_{m0}(y) - t_0$.

The derivative $\partial t_m / \partial y$ is equal to zero only at infinity for this solution.

In the zonal calculation, we employ formulas (1)–(5). We make several approximations with the use of formula (6) on each portion; as the initial air temperature we have taken

$$t_0 = \left| t_{in} + t_{out} \right| / 2\,, \quad (7)$$

where t_{in} and t_{out} are the inlet and outlet temperatures determined with account for the solution of the equation of heat exchange (1) and mass exchange (2). The length of the tracheal portion was taken to be 0.01 m; a twofold (or larger) further decrease in the portion length leads to a change of about five hundredths of a degree in the air and wall temperatures, which is by no means outside the computational accuracy. We set t_{m0} equal, for example, to 36.7°C irrespective of y in the first portion and for the first inhalation after the change of the breathing regime (flow rate, temperature, and moisture content of the inhaled air). We calculate the temperature distribution in the mass from for-



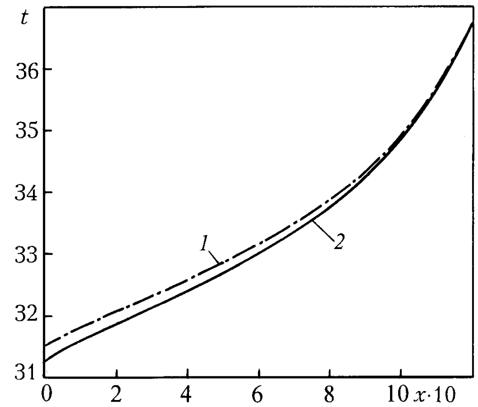


Fig. 3. Temperature distribution of air over the trachea length at an inlet temperature of $20^{o}C$ and a relative humidity of 76%: 1) the first interval of breathing for an instant of time of 1.2 sec from the beginning of inhalation; 2, 3, and 4) the tenth, twentieth, and thirtieth intervals of breathing for the same instant of time from the beginning of the preceding inhalation. t, ^{o}C; x, m.

Fig. 4. Air-temperature distribution over the trachea length in exhalation: 1) the tenth interval of breathing for an instant of time of 4.4 sec from the beginning of the preceding inhalation; 2) the same, the twentieth interval. t, ^{o}C; x, m.

mula (6) in reckoning the time from the beginning of the portion. Next, employing formulas (1), (2), and (7), we find a refined temperature of the air t_0 in the portion. In the continued approximations, we compare t_0 and the value obtained in the previous approximation. The calculation is stopped when the temperature coincides accurate to one hundredth of a degree. To calculate the next portion we take the distribution $t_{m0}(y)$ and t_{in} from the results of the calculations of the previous portion. In exhalation, the air coming into the trachea is assumed to be heated to body temperature ($36.7^{o}C$). The values of $t_{m0}(y)$ are taken from the results of the previous calculations. Analogously we carry out calculation for the subsequent inhalations and exhalations. We disregard the longitudinal flows of heat over the trachea wall, since the longitudinal temperature differences in the portions are small as compared to the transverse differences.

For the above value of the coefficient of heat transfer α in the trachea we have calculated (according to the algorithm described) the air and wall temperatures under different conditions. As an illustration, below we show the results of such calculations. Figure 2 gives the temperature distributions in the wall for the thirtieth interval of breathing (the time of inhalation and exhalation is taken as the breathing interval). The penetration of a temperature wave into the mass of the wall does not exceed 1 mm. The temperature of the wall at the beginning of the trachea, on the source side of the incoming air, decreases (since the incoming air has a lower temperature) during the inhalation and increases in exhalation. The situation was approximately the same in a qualitative sense for the previous intervals of breathing, and the penetration depth of a temperature wave did not increase, in practice, from interval to interval. In the initial intervals, the wall temperature was higher and it gradually decreased due to the cooling by the inhaled cold air. It should also be noted that the cooling in the initial tracheal portion, after the throat slit, can be larger than is taken in calculation as a consequence of the turbulization of the inhaled-air flow. Therefore, the temperature changes of the wall will be stronger but the estimated penetration of the thermal wave deep into the mass will not increase very substantially. We also note that the temperature of the trachea wall was higher in the initial intervals of breathing; therefore, the temperature of the air in inhalation increased more rapidly for the first interval of breathing than for the subsequent intervals (Fig. 3). The air temperatures in the trachea throughout its length differ little now for the tenth and twentieth intervals of breathing and they are nearly coincident for the twentieth and thirtieth intervals, which suggests the stabilization of the process of heating of the air. Figure 4 gives the change in the air temperature along the trachea length in exhalation. We can judge from the plot the character of cooling of the air exhaled by the trachea walls for different intervals of breathing. Figure 5a gives results of the calculations of the temperature walls when air

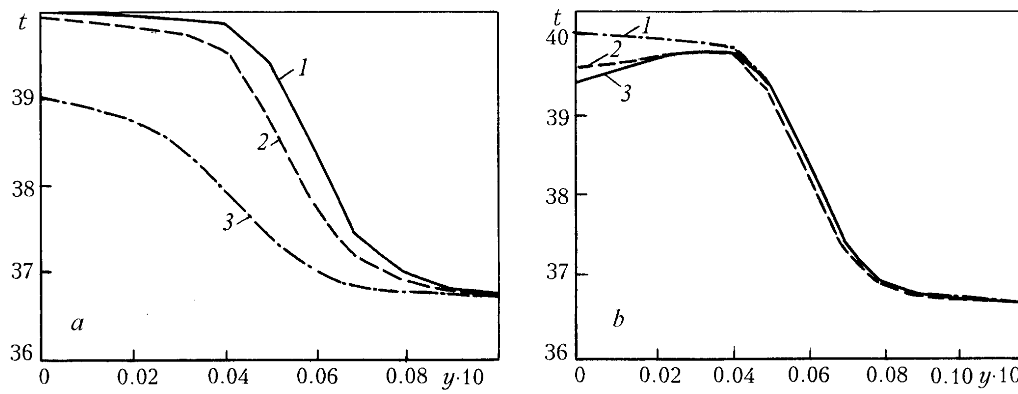


Fig. 5. Temperature distribution in the depth of the trachea wall at an inlet temperature of air of 40^{o}C and a relative humidity of 76%, which corresponds to the tenth interval of breathing: a) 0.2 sec from the beginning of inhalation [1, 2, and 3) distance $x = 1\cdot10^{-2}$, $6\cdot10^{-2}$, and $12\cdot10^{-2}$ m]; b) for the instants of time from the beginning of inhalation [1) after 0.2, 2) 1.2, and 3) 4.4 (exhalation)] at $x = 1\cdot10^{-2}$ m.

with a temperature of 40^{o}C and a relative humidity of 76% comes into the trachea. The given temperature distributions in the depth of the mass correspond to the tenth interval of breathing at different distances from the beginning of the trachea. Figure 5b gives the temperature distributions in the wall for several instants of time for the tenth interval of breathing, also.

We note that the calculation results given in the paper could also have been influenced by the transverse heat transfer in incomplete cartilaginous rings strengthening the trachea. However, these incomplete rings are comparatively thin (1.5 mm) and are spaced at 5 to 6 mm. Moreover, their thermal conductivity differs little from the thermal conductivity of the tissue. Thus, the transfer of heat in the incomplete cartilaginous rings is small, and they do not act as thermal fins.

In closing, we note that the transfer of heat with the blood flow (perfusion heat transfer) can also be of importance in the tissues of the body apart from heat conduction. However, here the blood flow in medium- and large-diameter blood vessels plays a dominant role [9, 10], while heat transfer with the blood flow is small. There are no large blood vessels at the trachea walls but there is a network of capillaries with a diameter of about 10^{-5} m and a velocity of motion of the blood of $(0.5–1.0)\cdot10^{-3}$ m/sec [11]. Perfusion heat transfer in such capillaries must be relatively small as compared to the thermal conductivity of the tissue. Nonetheless, calculations of the temperature of the trachea wall with an unattainably high value of the coefficient of thermal conductivity of the mass were performed for an obviously exceeded estimate. It is precisely the value of this coefficient that was taken to be tenfold higher than the value taken earlier (0.6 W/(m·deg)). According to the results of such calculations, when air with a temperature of 20^{o}C comes into the trachea, the wall temperature increases (by 3 to 4 degrees at the beginning of the trachea) because of the more intense supply of heat from the inside. However, the general form of the temperature curves remains the same and the depth of penetration of a temperature change into the wall has decreased by only 0.1–0.2 mm (for the thirtieth interval of breathing). Thus, the penetration depth is a rather stable quantity changing weakly with conditions. The calculation results given in the paper will little change under real changes in the coefficients of thermal conductivity of the mass and in the heat and mass transfer and the character of breathing (as opposed to that of Fig. 1). In a qualitative sense, the results obtained are of a general character. In the case of breathing under physical load, the processes of heat and mass exchange will be more intense, and rough calculations can be carried out according to the procedure presented in the paper.

NOTATION

$\rho C_p = \rho_v C_{pv} + \rho_{air} C_{pair}$, heat capacity per unit volume of moist air, J/(m^3·deg); ρ_v, ρ_{air}, and ρ, densities of the water vapor (steam) and of the dry and moist air, kg/m^3; C_{pv}, C_{pair}, and C_p, specific heats of the water vapor and the air and moist air, J/(m^3·deg); τ, time, sec; x, longitudinal coordinate, m; w, velocity of moist air, m/sec; $\vartheta = t - t_w$, ^{o}C; t, running temperature of moist air, ^{o}C; t_w, temperature of the interior surface of the channel wall, ^{o}C; $\alpha = \mathrm{Nu}\lambda / d$, heat-transfer coefficient, W/(m^2·deg); Nu, thermal Nusselt number; λ, thermal conductivity of moist air, W/(m·deg); d, channel diameter, m; $Y = P_v / P$, relative partial pressure of the vapor; $P = P_v + P_{air}$, total pressure of moist air, Pa; P_v and P_{air}, partial pressure of the vapor and dry air, Pa; $Y_w = R_{v,w} / P$; $R_{v,w}$, partial pressure of the saturated vapor at the surface temperature, Pa; $\beta = \mathrm{Nu}_D D / d$, mass-transfer coefficient, m/sec; Nu_D, diffusional Nusselt number; D, coefficient of diffusion of the water vapor in air, m^2/sec; $\mathrm{St}_{min} = \alpha / (\rho C_p w_{max})$, minimum value of the Stanton number, corresponding to the maximum velocity w_{max} in inhalation or exhalation; K, approximation coefficients in formula (5); $\varphi(\tau) = w(\tau) / w_{max}$; ν, internal variable in the integral in formulas (1) and (2), which corresponds to the time, sec; $\mathrm{St}_{D\min} = \beta / w_{max}$, minimum value of the diffusional Stanton number; Le = a/D, Lewis number; a, thermal diffusivity of moist air, m^2/sec; M_v, molecular mass of the vapor, kg/kmole; R, universal gas constant, J/(kmole·K); $\bar{T} = (T + T_w)/2$, average temperature of the boundary layer, K; r, latent heat of the water vapor–water phase transition, J/kg; y, transverse coordinate in the semiinfinite mass reckoned from the interior surface of the channel wall, m; λ_m and a_m, thermal conductivity and thermal diffusivity of the body mass respectively, W/(m·deg) and m^2/sec; η and ξ, internal variables in the integrals of formula (6), which correspond to the transverse coordinate, m. Subscripts: 0, initial value; v, vapor; air, air; w, wall; m, mass, min, minimum value; max, maximum value; D, differential; in, inlet; out, outlet.

REFERENCES

1. N. A. Belyakov, V. B. Serikov, and M. S. Rumm, *Biophysics*, **31**, No. 5, 901–907 (1986).
2. N. A. Belyakov, V. B. Serikov, and A. A. Loit, *Usp. Fiz. Nauk*, **19**, No. 2, 88–97 (1988).
3. V. B. Serikov, M. S. Rumm, G. L. Pasternak, and N. A. Belyakov, *Fiziol. Zh. SSSR*, **12**, No. 10, 1415–1418 (1986).
4. P. W. Scherer and L. M. Hanna, in: A. Shitzer and R. C. Eberhart (eds.), *Heat Transfer in Medicine and Biology*, Vol. 2, Kluwer Academic (1984), pp. 287–306.
5. A. G. Amelin, *Theoretical Principles of Formation of Mist in Vapor Condensation* [in Russian], Moscow (1966).
6. E. R. Weibel, *Morphometry of the Human Lung* [Russian translation], Moscow (1970).
7. C. E. Jonson, L. S. Linderoth, Jr., and L. M. Nuckols, *J. Biomech. Eng.*, February, 45–53 (1977).
8. V. I. Smirnov, *A Course in Higher Mathematics* [in Russian], Vol. 2, Leningrad (1948).
9. Z. P. Shul'man, B. M. Khusid, and I. V. Fain, *Inzh.-Fiz. Zh.*, **68**, No. 1, 75–85 (1995).
10. H. H. Pennes, *J. Appl. Physiol.*, **1**, 93–122 (1948).
11. V. A. Levtov, S. A. Regirer, and N. Kh. Shadrina, *Rheology of Blood* [in Russian], Moscow (1982).